S2 Normal Dist^n Past Qu.s

1) June 2004 no.1

The wingspan of shelduck is normally distributed with mean 121.5 cm and standard deviation 5.3 cm. Find the probability that one randomly chosen shelduck has a wingspan of at least 130 cm. [3]

2) June 2004 no. 5

The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X > 2\mu) = 0.0228$.

(i) Find the value of μ in terms of σ. [3]

In order to calculate the actual values of μ and σ, more information is required.

(ii) Explain why neither of the following extra pieces of information would enable you to work out the actual values of μ and σ:

(a) $P(X < 0) = 0.0228$; [1]

(b) $P(X < \mu) = 0.5$. [1]

(iii) Given that $P(X < 7.0) = 0.7881$, calculate the actual values of μ and σ. [4]

3) Jan 2005 no.3

The lifetime, T months, of properly made tap washers is modelled by a normal distribution with mean μ and standard deviation σ.

(i) It is given that $P(T > 80.0) = 0.05$ and $P(T < 70.0) = 0.75$. Find the values of μ and σ. [6]

(ii) Some tap washers are badly made and therefore have a very short lifetime. Give a reason why a normal distribution may not be a good model for the distribution of the lifetimes of all washers. [1]

4) June 2005 no.2

The continuous random variable Y is normally distributed with mean $17.0 + 2.0d$, where d can take different values. Thus, for example, if $d = 4.0$, the mean of Y is 25.0. The standard deviation of Y is 3.0 whatever the value of d.

(i) When $d = 5.0$, find $P(Y < 25.0)$. [3]

(ii) Find the value of d for which $P(Y < 40.0) = 0.975$. [4]

5) Jan 2006 no.5

In an investment model the increase, $Y\%$, in the value of an investment in one year is modelled as a continuous random variable with the distribution $N\left(\mu, \frac{1}{4}\mu^2\right)$. The value of μ depends on the type of investment chosen.

(i) Find $P(Y < 0)$, showing that it is independent of the value of μ. [4]

(ii) Given that $\mu = 6$, find the probability that $Y < 9$ in each of three randomly chosen years. [4]

(iii) Explain why the calculation in part **(ii)** might not be valid if applied to three consecutive years. [1]

6) June 2006 no.3

The continuous random variable T has mean μ and standard deviation σ. It is known that $P(T < 140) = 0.01$ and $P(T < 300) = 0.8$.

(i) Assuming that T is normally distributed, calculate the values of μ and σ. [6]

In fact, T represents the time, in minutes, taken by a randomly chosen runner in a public marathon, in which about 10% of runners took longer than 400 minutes.

(ii) State with a reason whether the mean of T would be higher than, equal to, or lower than the value calculated in part **(i)**. [2]

7) Jan 2007 no.1

The random variable H has the distribution $N(\mu, 5^2)$. It is given that $P(H < 22) = 0.242$. Find the value of μ. [4]

8) June 2007 no.4

X is a continuous random variable.

(i) State two conditions needed for X to be well modelled by a normal distribution. **[2]**

(ii) It is given that $X \sim N(50.0, 8^2)$. The mean of 20 random observations of X is denoted by $\bar{X}$. Find $P(\bar{X} > 47.0)$. **[4]**

9) Jan 2008 no.1

The random variable T is normally distributed with mean μ and standard deviation σ. It is given that $P(T > 80) = 0.05$ and $P(T > 50) = 0.75$. Find the values of μ and σ. [6]

10) June 2008 no. 2

The annual salaries of employees in a company have mean £30 000 and standard deviation £12 000.

(i) Assuming a normal distribution, calculate the probability that the salary of one randomly chosen employee lies between £20 000 and £24 000. [4]

(ii) The salary structure of the company is such that a small number of employees earn much higher salaries than the others. Explain what this suggests about the use of a normal distribution to model the data. [2]

11) June 2009 no.1

The random variable H has the distribution $N(\mu, \sigma^2)$. It is given that $P(H < 105.0) = 0.2420$ and $P(H > 110.0) = 0.6915$. Find the values of μ and σ, giving your answers to a suitable degree of accuracy. **[6]**

12) Jan 2010 no.6

The continuous random variable X has the distribution $N(\mu, \sigma^2)$.

(i) Each of the three following sets of probabilities is impossible. Give a reason in each case why the probabilities cannot both be correct. (You should not attempt to find μ or σ.)

(a) $P(X > 50) = 0.7$ and $P(X < 50) = 0.2$ **[1]**

(b) $P(X > 50) = 0.7$ and $P(X > 70) = 0.8$ **[1]**

(c) $P(X > 50) = 0.3$ and $P(X < 70) = 0.3$ **[1]**

(ii) Given that $P(X > 50) = 0.7$ and $P(X < 70) = 0.7$, find the values of μ and σ. **[4]**

13) June 2010 no.3

Tennis balls are dropped from a standard height, and the height of bounce, H cm, is measured. H is a random variable with the distribution $N(40, \sigma^2)$. It is given that $P(H < 32) = 0.2$.

(i) Find the value of σ. **[3]**

(ii) 90 tennis balls are selected at random. Use an appropriate approximation to find the probability that more than 19 have $H < 32$. **[6]**

14) June 2011 no.2

The random variable Y has the distribution $N(\mu, \sigma^2)$. It is given that

$$P(Y < 48.0) = P(Y > 57.0) = 0.0668.$$

Find the value y_0 such that $P(Y > y_0) = 0.05$. **[7]**

15) Jan 2012 no.3

The random variable G has a normal distribution. It is known that

$$P(G < 56.2) = P(G > 63.8) = 0.1.$$

Find $P(G > 65)$. **[6]**

16) June 2012 no. 6

At a tourist car park, a survey is made of the regions from which cars come.

(i) It is given that 40% of cars come from the London region. Use a suitable approximation to find the probability that, in a random sample of 32 cars, more than 17 come from the London region. Justify your approximation. **[7]**

(ii) It is given that 1% of cars come from France. Use a suitable approximation to find the probability that, in a random sample of 90 cars, exactly 3 come from France. **[4]**